

(3 Hours)

(Total Marks : 80)

Note: 1. Question no. 1 is compulsory.

2. Attempt any **three** questions out of remaining **five** questions.

Q.1.[a] Evaluate $L[\sin 2t \cos t \cosh 2t]$. [5]

[b] How many friends must you have to guarantee that atleast five of them have birthday in the same month. [5]

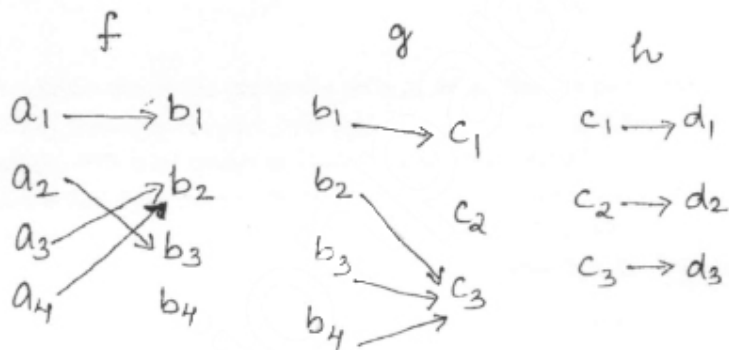
[c] Determine the constants a, b, c, d, e so that the function $f(z) = ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2 + i(4x^3y - exy^3 + 4xy)$ is analytic. [5]

[d] Out of one lakh people 51500 are female and 48500 are male. Among the females 9000 are singers, among the males 30200 are singers. A person chosen randomly. If A, B, C are the events that a singer is chosen, a female is chosen and male is chosen respectively then find (i) $P(A/B)$ (ii) $P(A/C)$ (iii) $\frac{P(A \cap C)}{P(B/A)}$ (iv) $P(C/A)$. [5]

Q.2. [a] Using Venn diagram show that $P \cap (Q \oplus R) = (P \cap Q) \oplus (P \cap R)$. [6]

[b] Evaluate $L\{f(t)\}$ where $f(t) = \begin{cases} 1 & 0 \leq t < a \\ -1 & a < t < 2a \end{cases}$ and $f(t+2a) = f(t)$. [6]

[c] Let f, g, h be the functions shown in the diagram : [8]



Find : (i) $g \circ f$, $h \circ (g \circ f)$, $(h \circ g) \circ f$, h^{-1}

(ii) Identify onto and one-one function for 3 of them.

Q.3. [a] Find analytic function $f(z) = u + iv$ where $v = \frac{x}{x^2 + y^2} + \cosh x \cos y$. [6]

[b] Solve $(D^2 + 2D + 5)y = e^{-t} \sin t$, when $y(0) = 0$, $y'(0) = 1$. [6]

[c] Evaluate (i) $L\left\{\frac{1}{t}(1 - \cos t)\right\}$ [8]

(ii) $\int_0^{\infty} e^{-t} \left(\int_0^t u^4 \sinh u \cosh u \, du \right) dt$

- Q.4. [a]** Evaluate using convolution theorem $L^{-1}\left[\frac{(s+2)}{(s^2+4s+8)^2}\right]$ [6]
- [b]** Find bilinear transformation which maps the points $z = -1, 1, \infty$ onto $w = -i, -1, i$. [6]
- [c]** Three machines A, B and C produce respectively 25%, 35% and 40% of the total number of items of a factory. The percentages of defective output of these machines are respectively 5%, 4% and 2%. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine A. [8]
- Q.5. [a]** Suppose repetitions are not permitted. [6]
- (i) How many four-digit numbers can be formed from the digits 1, 2, 3, 5, 7, 8?
- (ii) How many of the numbers in part (a) are less than 4000?
- (iii) How many of the numbers in part (a) are multiples of 5?
- [b]** Let $A = \{1, 2, 3, 4, 12\}$ and let R be the relation on A defined by xRy if and only if "x divides y", Show that (A, R) is a PO set. Draw the diagram of R. [6]
- [c]** Evaluate (i) $L^{-1}\left[\frac{e^{-5s}}{(s-2)^4}\right]$ (ii) $L^{-1}\left[\log\left(\frac{s+3}{s+5}\right)\right]$ [8]
- Q.6. [a]** It is known that at the university 60% of the professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 30% play tennis and jog, 40% play bridge and jog. If someone claimed that 20% of the professors jog and play bridge and tennis, would you believe this claim? Why? [6]
- [b]** Solve $a_{r+2} + 2a_{r-1} - 3a_r = 0$ that satisfies $a_0 = 1, a_1 = 2$. [6]
- [c]** (i) If $f(z)$ is an analytic and $|f(z)|$ is constant, show that $f(z)$ is constant. [8]
- (ii) Find the image of $|z-a| = a$ under the transformation $w = \frac{1}{z}$.